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HEAT TRANSFER IN THE THERMAL ENTRY LENGTH WITH LAMINAR FLOW IN AN ANNULUS

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Abstract—The solution to the Graetz problem in an annulus with a heated core and an insulated outer wall is presented. Both uniform temperature and uniform heat input on the inside wall are considered for a number of arbitrarily chosen radius ratios. The case of parallel plates with one side insulated is included for comparison.

The solution is extended to certain cases of axial variation of the core heat input and to the case in which heat is transferred at uniform but unequal rates at both walls.

NOMENCLATURE

- C_p, constant pressure specific heat;
- C, constant in the solution with the inside heated;
- d, mean diameter defined as 4 × area/perimeter;
- D, constant in the solution with the outside heated;
- G, fully developed temperature with inside heated;
- H, fully developed temperature with outside heated;
- k, thermal conductivity;
- L, dimensionless passage length;
- q, heat transfer rate per unit area;
- r, radius;
- R, dimensionless radius;
- t, temperature;
- T, dimensionless temperature in the constant wall temperature case;
- u, velocity;
- U, dimensionless velocity;
- x, distance from the entrance;
- Y, eigenfunction of equation (5) or (11) in the solution for the heating on the inside;
- Y', slope of eigenfunction;
- Z, eigenfunction of equation (11) in the solution for heating on the outside;
- y, dimensionless distance from the wall in a parallel passage;
- y', distance from one wall;
- y_0, distance between the walls in a parallel passage;
- Re, Reynolds number;
- Pr, Prandtl number;
- Nu, Nusselt number.

Greek symbols

- α, thermal diffusivity;
- λ, eigenvalue;
- θ, dimensionless temperature in the constant heat input case;
- ρ, density;
- ν, kinematic viscosity.

Suffixes

- 1, fully developed;
- 2, in the thermal entrance region;
- e, at the inlet;
- i, at the inner surface of the annulus;
- m, bulk mean value;
- max, at the position of maximum velocity;
- n, eigenvalue number;
- o, at the outer surface of the annulus.

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INTRODUCTION

THE solution of the heat-transfer problem with fully developed laminar flow through a circular tube with the wall at constant temperature has been the subject of a great deal of attention in the past. The problem was solved in principle by Graetz [1] but the numerical calculation of the required eigenvalues was very tedious and he was able to obtain only the first three. Nusselt [2] improved the accuracy of the calculation, again of the first three eigenvalues. Sellars *et al.* [3] extended the analysis to the cases of constant heat input and axial variations of temperature and heat input and also derived asymptotic expressions for the higher eigenvalues.

Recently, Brown [4] applied a digital computer to the Graetz problem and obtained the first ten eigenvalues for both a round tube and a flat duct. The amount of numerical work involved in these calculations is so large that the use of a computer is essential to obtain adequate accuracy.

The solution of the equivalent problem in an annulus follows the Graetz solution very closely. There is the additional variable of the radius ratio and since there are two surfaces involved there are many further possible boundary conditions. Previous work on this problem is very limited. Jakob and Rees [5] gave results for the fully developed temperature profile but did not quote the Nusselt numbers since they did not calculate the bulk mean temperatures. Murakawa [6] presented an analysis for the combined hydrodynamic and thermal entry length, but this is very complicated and numerical results were quoted for one radius ratio only. He also gave results for the thermal entry length for a particular radius ratio, but this does not appear to agree with the corresponding values given in this paper. Montgomery and Weiss [7] have surveyed the work on heat transfer in non-circular ducts and include a useful bibliography.

Results are presented in this paper for the thermal entry region with laminar flow in an annulus with the outside wall insulated. This case has some practical bearing in double pipe heat exchangers. Both constant temperature and constant heat input on the inside pipe are considered. The results can be easily extended to axial variations of temperature and heat input on the inner wall by the superposition thereon. The case of unequal heat inputs on each side may also occur in practice and this boundary condition is briefly considered.

BASIC EQUATIONS

The following assumptions are made.

(1) Constant fluid physical properties.
(2) Fully developed laminar flow throughout.
(3) Heat flow in the axial direction is negligible.
(4) Temperature changes due to dissipative effects are negligible.
(5) Uniform temperature in the fluid at the start of heating.

The energy equation then becomes

$$\rho \, . \, C_p \, . \, u \frac{\partial t}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r}\left(\alpha \, . \, r \, . \, \frac{\partial t}{\partial r}\right). \tag{1}$$

The following dimensionless parameters are defined

$$U = \frac{u}{u_m}, \qquad R = \frac{r}{r_o - r_i} = \frac{2r}{d}$$

$$L = \frac{4}{Re\,Pr}\left(\frac{x}{d}\right), \qquad Re = \frac{u_m d}{\nu}.$$

For an annulus we have [8]

$$U = \frac{2(R_o^2 - R_i^2 - 2R_{\max}^2 \ln R_o/R_i)}{R_o^2 + R_i^2 - 2R_{\max}^2} \tag{2}$$

where

$$R_{\max}^2 = \frac{R_o^2 - R_i^2}{2 \ln R_o/R_i}.$$

The bulk mean temperature t_m is defined

$$t_m = \frac{\int_{r_i}^{r_o} u \, . \, t \, . \, 2\pi r \, . \, \mathrm{d}r}{u_m \pi (r_o^2 - r_i^2)}$$

$$= \frac{2}{2R_i + 1} \int_{R_i}^{R_o} U \, . \, t \, . \, R \, . \, \mathrm{d}R. \tag{3}$$

Solution with uniform temperature on the inside wall, outside wall insulated

The dimensionless temperature T is defined by

$$T = \frac{t - t_i}{t_e - t_i}$$

the solution is

$$T = \sum_{n=0}^{\infty} C_n Y_n \exp(-\lambda_n^2 L). \tag{4}$$

Where λ_n and Y_n are the n^{th} eigenvalue and function of the Sturm Liouville problem:

$$\left.\begin{aligned} &\frac{\mathrm{d}^2 Y}{\mathrm{d}R^2} + \frac{1}{R}\frac{\mathrm{d}Y}{\mathrm{d}R} + \lambda^2 U Y = 0 \\ &Y = 0, \qquad R = R_i \\ &\frac{\mathrm{d}Y}{\mathrm{d}R} = 0, \qquad R = R_o. \end{aligned}\right\} \tag{5}$$

The constants C_n are given by

$$C_n = \frac{\int_{R_i}^{R_o} R \,.\, U \,.\, Y_n \,.\, \mathrm{d}R}{\int_{R_i}^{R_o} R \,.\, U \,.\, Y_n^2 \,.\, \mathrm{d}R}. \tag{6}$$

The Nusselt number is

$$Nu = \frac{2R_i + 1}{R_i} \frac{\sum_{n=0}^{\infty} C_n Y'_{ni} \exp\left(-\lambda_n^2 L\right)}{\sum_{n=0}^{\infty} \frac{C_n Y'_{ni}}{\lambda_n^2} \exp\left(-\lambda_n^2 L\right)}. \tag{7}$$

The eigenvalues and constants were calculated with the aid of the Manchester University Mercury Computer for the annuli shown in Fig. 1 and the results are given in Table 1. The fully developed Nusselt number is shown on Fig. 2 and the values in the entrance region on Fig. 3.

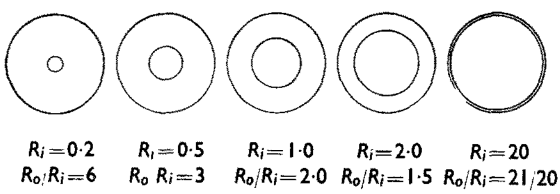


FIG. 1. The annuli.

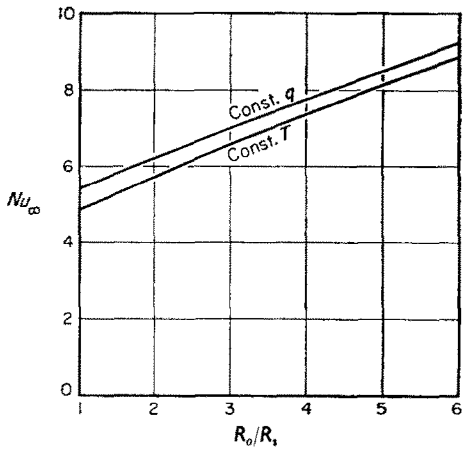


FIG. 2. Fully developed Nusselt numbers.

As a check, the values for a parallel wall duct were calculated. The governing equation was separated in a slightly different way and the expression for Nusselt number was obtained in the following form

$$Nu = \frac{\sum_{n=0}^{\infty} C_n Y'_{ni} \exp\left(-\lambda_n^2 L/6\right)}{3 \sum_{n=0}^{\infty} \frac{C_n Y'_{ni}}{\lambda_n^2} \exp\left(-\lambda_n^2 L/6\right)}. \tag{8}$$

The relevant values are given in Table 2. The Nusselt numbers are very close to those for the annulus with radius ratio 21:20 and could not be shown separately on Fig. 3. For a direct comparison between the eigenvalues for the parallel plate case and those for the annuli it is necessary to divide the former by $\sqrt{6}$. The dotted curve is taken from Prins *et al.* [9] and is for both sides at uniform temperature.

Table 1. Eigenvalues and constants for the annulus with uniform inside wall temperature

No.	$R_i = 0{\cdot}2$ λ_n	$R_i = 0{\cdot}2$ $C_n Y'_{ni}$	$R_i = 0{\cdot}5$ λ_n	$R_i = 0{\cdot}5$ $C_n Y'_{ni}$	$R_i = 1{\cdot}0$ λ_n	$R_i = 1{\cdot}0$ $C_n Y'_{ni}$	$R_i = 2{\cdot}0$ λ_n	$R_i = 2{\cdot}0$ $C_n Y'_{ni}$	$R_i = 20{\cdot}0$ λ_n	$R_i = 20{\cdot}0$ $C_n Y'_{ni}$
1	1·124954	0·8497293	1·282161	1·543650	1·383003	2·651176	1·456400	4·838529	1·546883	44·02721
2	4·582686	0·3631899	4 704357	0·8017342	4·767710	1·522857	4·807803	2·956594	4·851740	28·65738
3	7 830728	0·2911149	7·981437	0·6568992	8·052764	1·259203	8·093105	2·457434	8·130398	23·94991
4	11 04763	0·2554939	11·23569	0·5815754	11 32103	1·118297	11·36629	2·186027	11·40302	21·34054
5	14·25304	0·2328140	14·48154	0·5324168	14·58304	1 025315	14·63486	2·005779	14·67321	19·59566
6	17·45278	0·2165746	17 72327	0·4966960	17·84195	0·9573533	17·90112	1·873612	17·94215	18·31187
7	20·64929	0·2041237	20·96265	0·4690400	21·09910	0·9045479	21·16606	1 770727	21·21035	17·31062
8	23·84375	0·1941389	24·20056	0·4467082	24·35514	0·8618086	24·43017	1·687354	24·47810	16·49834
9	27·03685	0·1858733	27·43748	0·4281261	27 61044	0·8261867	27·69372	1·617808	27·74554	15·82019
10	30·22899	0·1788674	30 67373	0 4123141	30 86523	0 7958358	30·95689	1·558519	31·01279	15·24163

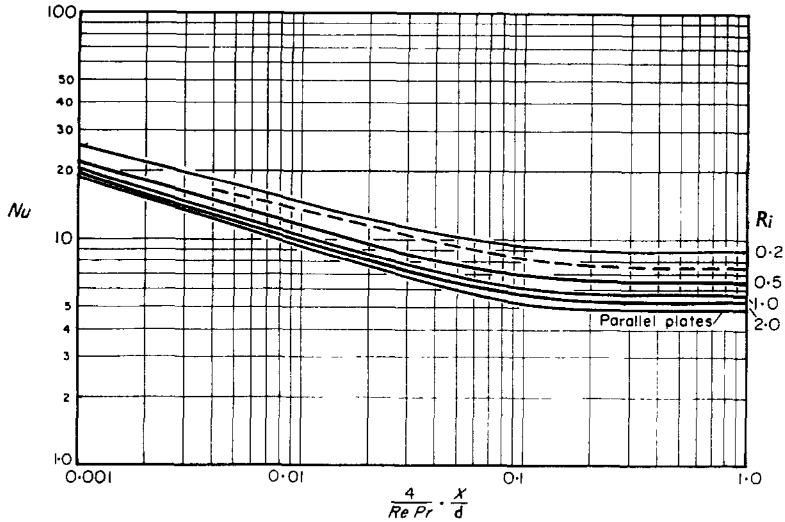


FIG. 3. Nusselt numbers for the uniform temperature case. (Both sides heated [9]).

Table 2. Eigenvalues and constants for the parallel plate case with uniform temperature

No.	λ_n	$C_n Y'_{ni}$
1	3·818667	2·176545
2	11·89723	1·427232
3	19·92414	1·193603
4	27·93835	1·063782
5	35·94734	0·9768908
6	43·95364	0·9129374
7	51·95837	0·8630460
8	59·96211	0·8225616
9	67·96519	0·7887606
10	75·96784	0·7599269

Solution with uniform heat input on the inside wall, outside wall insulated

Sellars *et al.* [3] showed that the results for this case can be derived from those for uniform temperature by the method of Laplace transforms. The result is obtained in the form of a power series whose roots are identically the eigenvalues of the uniform heat input case. Both this method and the method of direct integration were used initially to obtain these eigenvalues. However, the method of Sellars *et al.* does require a large number of uniform temperature eigenvalues to derive the uniform heat input eigenvalues, and from the first ten previously calculated, the values obtained did not compare sufficiently well. The direct integration of the equations was therefore again used to calculate this case.

It is more convenient with uniform heat input to define the dimensionless temperature θ so that $\theta = (t - t_e)/(qd/k)$.

The usual procedure (see, for example, Sparrow *et al.* [10]) is followed of dividing the solution into two parts, θ_1 the fully developed solution, and θ_2 an entry temperature which disappears at large L.

The solution is written

$$\theta_1 = \frac{R_i L}{2R_i + 1} + G \tag{9}$$

where G is a function of R only and is the fully developed profile having a bulk mean value of zero

$$\theta_2 = \sum_{n=0}^{\infty} C_n Y_n \exp(-\lambda_n^2 L). \tag{10}$$

Where λ_n and Y_n are the eigenvalues and functions of

$$\left.\begin{aligned} &\frac{d^2 Y}{dR^2} + \frac{1}{R}\frac{dY}{dR} + \lambda^2 U Y = 0 \\ &\frac{dY}{dR} = 0, \qquad R = R_i \\ &\frac{dY}{dR} = 0, \qquad R = R_o \end{aligned}\right\} \tag{11}$$

and

$$C_n = - \frac{\int_{R}^{R_o} R \,.\, U \,.\, G \,.\, Y_n \,\mathrm{d}R}{\int_{R_i}^{R_o} R \,.\, U \,.\, Y_n^2 \,\mathrm{d}R}. \tag{12}$$

The Nusselt number becomes

$$Nu = \frac{1}{G_i \left[1 - \sum_{n=0}^{\infty} C_n \exp\left(- \lambda_n^2 L\right)\right]}. \tag{13}$$

The eigenvalues and constants are given in Table 3 and the entrance region Nusselt number is shown in Fig. 4.

Again the parallel plate case was used for comparison and the values are given in Table 4.

The fully developed temperature profile with parallel walls is easily found to be

$$G = y^3 - \frac{y^4}{2} - y + \tfrac{13}{35} \tag{14}$$

where $y = y'/y_0$.

Also the Nusselt number

$$Nu = \frac{2}{\frac{13}{35} \left[1 - \sum_{n=0}^{\infty} C_n \exp\left(- \lambda_n^2 L/6\right)\right]}. \tag{15}$$

Table 3. Eigenvalues and constants for the annulus with uniform heat input

No.	$R_i = 0{\cdot}2$ λ_n	C_n	$R_i = 0{\cdot}5$ λ_n	C_n	$R_i = 1{\cdot}0$ λ_n	C_n	$R_i = 20{\cdot}0$ λ_n	C_n
1	3·759784	0·3548737	3·728149	0·4024391	3·710159	0 4243879	3·695862	0·4459568
2	6·962990	0·1456844	6·985235	0·1493154	6·994383	0·1496722	7·001240	0·1495232
3	10·15092	0·08324779	10·22638	0·08121473	10·25972	0·07966408	10·28435	0·07794680
4	13·33620	0·05509405	13·46328	0·05217145	13·51925	0·05055061	13·56024	0·04887533
5	16·52106	0·03968200	16·69850	0·03682774	16·77621	0·03539872	16·83282	0·03396640
6	19·70603	0·03020993	19·93290	0·02762987	20·03179	0·02640702	20·10357	0·02519695
7	22·89124	0·02391886	23·16685	0·02163319	23·28655	0·02058747	23·37320	0·01956512
8	26·07671	0·01950030	26·40055	0·01748174	26·54078	0·01658108	26·64210	0·01570465
9	29·26243	0·01626300	29 63410	0·01447512	29·79465	0·01369226	29·91052	0·01293594
10	32·44839	0·01381129	32·86757	0 01221967	33·04830	0·01153305	33·17858	0·01087097
Value	$G_i = 0{\cdot}108469$		$G_i = 0{\cdot}143261$		$G_i = 0{\cdot}161785$		$G_i = 0{\cdot}184451$	

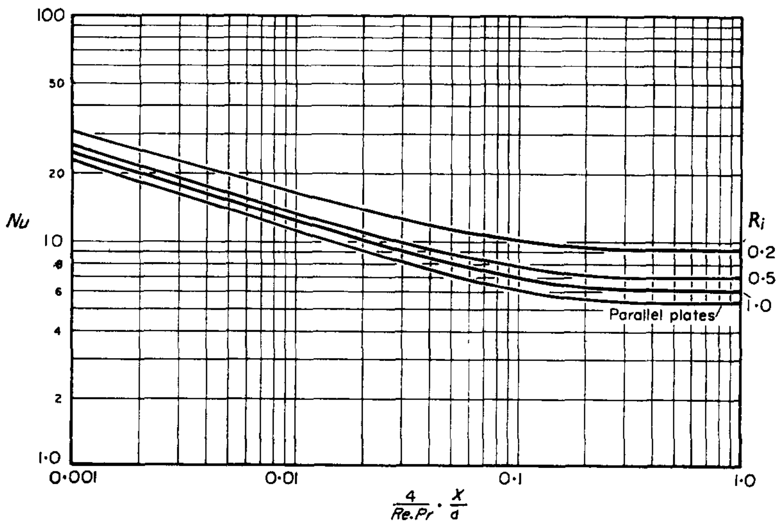


FIG. 4. Nusselt numbers for the uniform heat input case.

Table 4. Eigenvalues and constants for the parallel plate case with uniform heat input

No.	λ_n	C_n
1	9·052444	0·447007
2	17·14890	0·1495767
3	25·19075	0·0778954
4	33·21491	0·04881937
5	41·23094	0·03391114
6	49·24249	0·02515193
7	57·25131	0·01952389
8	65·25834	0·01567085
9	73·26415	0·01290481
10	81·26912	0·01084480

AXIAL VARIATIONS ON THE INSIDE WALL, OUTSIDE WALL INSULATED

Using Duhamel's integral, expressions can be obtained for Nusselt numbers with axial variations of temperature and heat input on the core in terms of the eigenvalues and constants already obtained [3].

(a) *Linear temperature increase*

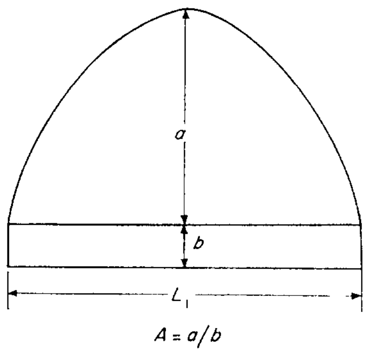
If the initial temperature is zero we obtain

$$Nu = \frac{2R_i + 1}{2} \frac{\sum\limits_{n=0}^{\infty} \frac{C_n Y'_{ni}}{\lambda_n^2}[1 - \exp(-\lambda_n^2 L)]}{\sum\limits_{n=0}^{\infty} \frac{C_n Y'_{ni}}{\lambda_n^4}[1 - \exp(-\lambda_n^2 L)]}. \quad (16)$$

The fully developed Nusselt numbers given by both equations (16) and (17) are equal to the uniform heat input value. The entrance lengths are rather longer than that for uniform input.

(c) *Half-sine wave heat input variation superimposed on a uniform heat input*

An axial variation of this form occurs in the fuel rod of a gas-cooled reactor, and although the flow in practice is usually turbulent, the expression for Nusselt number variation may be of interest. It is necessary to define the heated length over which the sine wave extends and the ratio A of the maximum output of the sine component to the output of the uniform component



Heat input variation.

The expression for Nusselt number is

$$Nu = \frac{1 + A \sin \frac{\pi L}{L_1}}{G_i\left\{1 - \sum\limits_{n=0}^{\infty} C_n \exp(-\lambda_n^2 L) + A \sin \frac{\pi L}{L_1} - A \sum\limits_{n=0}^{\infty} \frac{C_n \pi / L_1}{(\pi/L_1)^2 + \lambda_n^4}\left[\lambda_n^2 \cos \frac{\pi L}{L_1} + \frac{\pi}{L_1} \sin \frac{\pi L}{L_1} - \lambda_n^2 \exp(-\lambda_n^2 L)\right]\right\}} \quad (18)$$

(b) *Linear increase of heat input rate*

$$Nu = \frac{1}{G_i\left\{1 - \sum\limits_{n=0}^{\infty} \frac{C_n}{L \cdot \lambda_n^2}[1 - \exp(-\lambda_n^2 L)]\right\}}. \quad (17)$$

Figure 5 shows an example for the annulus with $R_i = 1{\cdot}0$. The value of L_1 was chosen arbitrarily at 0·1 and three values of the ratio A are shown. For large A the result tends to that for the pure half sine-wave and for small A to the uniform heat input case previously obtained.

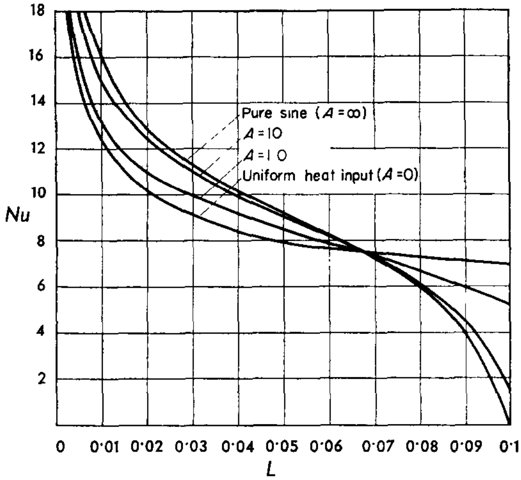


FIG. 5. Nusselt numbers: sine and step axial variation of heat input, $R_i = 1{\cdot}0$, $L_1 = 0{\cdot}1$.

UNIFORM HEAT INPUTS ON BOTH WALLS

In some practical situations heat transfer may occur on the outer wall. There may, for example, be a heat loss from the outer surface in a double-pipe heat exchanger.

With uniform but unequal heat transfers on each wall the same eigenvalues apply but a new set of constants must be determined. For the case of uniform but unequal temperatures it is necessary to calculate a new set of eigenvalues.

The result for unequal heat inputs is easily obtained by superposing the temperature profiles resulting from heating on the inside only and from heating on the outside only. In both parts of the solution the entrance component is represented by equation (11). It is necessary to add to the previous solution the fully developed profile with the outside heated and to modify the constants of equation (12). If, for this component, the developed temperature is H, the constants D_n and the functions Z_n the Nusselt numbers may be written:

Table 5 gives the additional constants which appear in the above expressions for the annulus having $R_i = 1{\cdot}0$ and Fig. 6 shows the Nusselt numbers for the particular case of $q_i = -q_o$ i.e. equal heat input rates on each side. The sign convention adopted for q_i and q_o in the above

Table 5. Constants required for calculating Nusselt numbers with unequal uniform heat inputs and uniform heat input on the outside only ($R_i = 1{\cdot}0$)

No.	Y_{no}	D_n	$(D_n Z_n)_i$
1	0·1308397	0·452343	− 0·1110536
2	− 0·1269216	0·1501202	0·0379933
3	0·1251726	0·0777155	− 0·0199435
4	− 0·1241494	0·0485114	0·0125517
5	0·1234661	0·0335976	− 0·00874106
6	− 0·1229720	0·0248635	0·00649469
7	0·1225951	0·01926552	− 0·00504785
8	− 0·1222961	0·01544087	0·00405556
9	0·1220517	0·0127001	− 0·00334231
10	− 0·1218466	0·01066189	0·00281051

$G_i = 0{\cdot}161785$
$G_o = 0{\cdot}042757$
$H_o = 0{\cdot}198548$
$H_i = 0{\cdot}0855143$

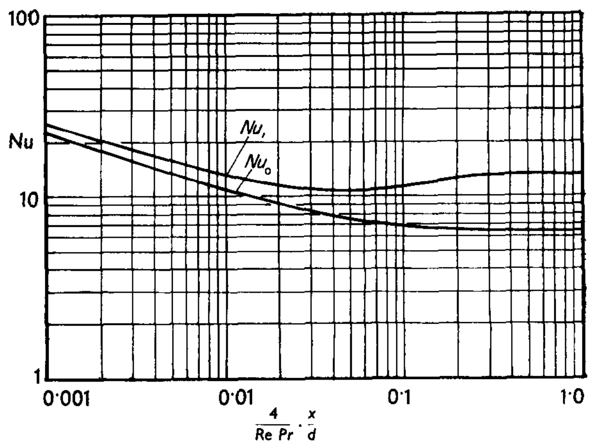


FIG. 6. Nusselt numbers: equal uniform heat input rate on each side, $R_i = 1{\cdot}0$.

$$Nu_i = \frac{1}{G_i[1 - \sum_{n=0}^{\infty} C_n \exp(-\lambda_n^2 L)] + \frac{q_o}{q_i}[H_i + \sum_{n=0}^{\infty} D_n Z_{ni} \exp(-\lambda_n^2 L)]}. \tag{19}$$

$$Nu_o = \frac{1}{\frac{q_i}{q_o}[G_o + \sum_{n=0}^{\infty} C_n Y_{no} \exp(-\lambda_n^2 L)] + H_o[1 - \sum_{n=0}^{\infty} D_n \exp(-\lambda_n^2 L)]}. \tag{20}$$

expression is that they have like signs when they are in the same direction.

Because of the form of equations (19) and (20) it is possible for the Nusselt number to behave in an unusual way. It may be negative or pass through an infinite value for certain ratios of q_o/q_i.

The wall temperature variation in this case can be inferred from the Nusselt number variation which yields the difference between the wall and bulk mean values. For unequal heat rates the bulk temperature is given by

$$t_m - t_e = \frac{dL}{k}\left\{q_i \frac{R_i}{2R_i + 1} - q_o \frac{R_o}{2R_o + 1}\right\}. \quad (21)$$

Thus the wall temperature variation on each side can be derived from (19) or (20) and (21).

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Résumé—La solution du problème de Graetz est étudiée dans le cas d'un anneau chauffé par un enroulement et dont la paroi extérieure est isolée.

On considère que l'on à la fois a une température uniforme et un flux de chaleur uniforme sur la paroi intérieure, pour des rapports de rayons arbitrairement choisis. On a étudié à titre de comparaison le cas de plaques planes parellèles isolées d'un côté.

La solution est étendue à certains cas de variation axiale du flux chaleur de l'enroulement et au cas où le flux de chaleur transmis aux deux parois est uniforme mais différent.

Zusammenfassung—Die Lösung des Graetz-Nusselt-Problems im Ringraum mit beheiztem Kern und isolierter Aussenwand wird angegeben. Sowohl der Fall konstanter Wandtemperatur als auch konstanter Wärmestromdichte wurde für eine Anzahl willkürlich gewählter Radienverhältnisse berücksichtigt. Als Vergleich dienen zwei parallele Platten, wovon eine isoliert ist. Die Lösung lässt sich auf einige achsiale Variationen der Heizleistung und auf den Fall konstanter, aber ungleicher Wärmezufuhr von beiden Wänden ausdehnen.

Аннотация—Дается решение задачи Гретца для изолированного снаружи кольцевого канала с внутренним источником тепла. Рассмотрен как случай постоянной температуры внутренней стенки, так и случай равномерной подачи тепла к ней для ряда произвольно выбранных отношений радиусов. Проведено сравнение с параллельными пластинами, одна поверхность которых изолирована.

Решение распространяется на определенные случаи аксиального изменения подачи тепла к центральной части канала и на случай, когда передача тепла к обеим стенкам происходит равномерно, но с разной скоростью.